Attenuation anisotropy linked to velocity anisotropy: theory and ultrasonic experiment

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Summary

We perform an ultrasonic experiment on anisotropy of wave propagation in the model constructed from plexiglas plates, with oil saturation and without any saturation. We determine the directional dependencies of P- and S-waves attenuations and velocities and estimate Thomsen-style parameters for attenuation $\varepsilon_0$, $\delta_0$, and $\gamma_0$ as well as Thomsen’s velocity parameters. We develop a theory that predicts certain relationship between anisotropies of P-, SH- and SV-wave attenuation and velocity in transversely isotropic (TI) media due to aligned fracture set. Taking into account the interplay of the velocity and attenuation anisotropies, we develop the effective TI model of the attenuative fractured medium fitted to the experimental data. We derive formulas for the parameters $\varepsilon_0$, $\delta_0$, and $\gamma_0$ in terms of complex-valued weaknesses that relates these parameters to fractured medium parameters. We found out that the ratio of the symmetry-axis P- and S-wave attenuations $Q_\theta/Q_\phi$ and the parameter $\delta_0$ are interlinked and strongly dependent of the fluid type in cracks that is confirmed by the experiment. The value of $\delta_0$ is much greater for dry-crack medium than for wet-crack medium, that is correlated with the $Q_\theta/Q_\phi$-ratio, which is on the contrary larger in wet-crack medium and smaller in dry-crack medium. This is in accordance with laboratory measurements at ultrasonic frequencies of P- and S-wave quality factors.

Introduction

Nowadays reliable seismic data on Q-anisotropy is in very short supply; there are only few recent works, e.g., Vasconselos & Jenner (2005), Maultzsch et al. (2005), and Varela et al. (2006). As for ultrasonic laboratory data, there are some petrophysical experiments on Q-anisotropy, where attenuation of P- and S-waves is measured at two orthogonal directions (e.g., Yin and Nur, 1992; Prasad and Nur, 2003; Best, 1994). In order to obtain the reliable estimates of Q-anisotropy for attenuative TI medium, one needs to know the values of P-, SV-, and SH-wave attenuations in the full interval of incidence angles, i.e., from 0 to 90° with the symmetry axis. There are few ultrasonic experiments with artificial plate models on P-wave attenuation anisotropy for wide range of propagation angles (e.g., Hosten et al., 1987; Zhu et al., 2007). Ultrasonic experiments on Q-anisotropy for all three types of waves, P, SV, and SH, in the full interval of wave incidence angles in TI medium are close to be absent.

In the presented experiment we get attenuations and velocities for P, SV-, and SH-waves. Joint analysis of the data for P-, SV-, and SH-wave attenuations and velocities provides the information for the proper reconstruction of the attenuation-anisotropy properties of the model. Join inversion of all the attenuations and velocities enables estimating Thomsen-style (Thomsen, 1986) parameters for attenuation $\varepsilon_0$, $\delta_0$, and $\gamma_0$, which were introduced by Zhu & Tsvankin (2006). We derive formulas for the directionally dependent attenuation and the attenuation parameters in terms of complex-valued weaknesses that relate these parameters to fractured-medium properties (Chichinina et al., 2004; 2005; 2006a; 2006b). We are going to reveal the new sense of the parameter $\delta_0$, which is close to be meaningful for fracture characterization in seismic exploration.

Theoretical background

To describe anisotropy in attenuative TI medium (due to a single set of fractures), we introduce complex-valued stiffness matrix. We rewrite the elastic stiffness matrix $\{C\}$ given in the form of Schoenberg and Sayers (1995, p. 207) and get the following complex-valued stiffness matrix $\{\tilde{C}\}$ for TI medium with vertical symmetry axis (VTI):

$$
\tilde{C}_i = \begin{bmatrix}
L & K & P & 0 & 0 & 0 \\
K & L & P & 0 & 0 & 0 \\
P & P & J & 0 & 0 & 0 \\
0 & 0 & 0 & N & 0 & 0 \\
0 & 0 & 0 & 0 & N & 0 \\
0 & 0 & 0 & 0 & 0 & R
\end{bmatrix}
\begin{bmatrix}
\tilde{\Delta}_N \\
\tilde{\Delta}_S \\
\tilde{\Delta}_P \\
\tilde{\Delta}_S \\
\tilde{\Delta}_S \\
\tilde{\Delta}_R
\end{bmatrix}
= \begin{bmatrix}
\lambda & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 & 0 & 0 \\
0 & 0 & \mu & 0 & 0 & 0 \\
0 & 0 & 0 & N & 0 & 0 \\
0 & 0 & 0 & 0 & N & 0 \\
0 & 0 & 0 & 0 & 0 & R
\end{bmatrix}
\begin{bmatrix}
\lambda \tilde{\Delta}_N \\
\lambda \tilde{\Delta}_S \\
\mu \tilde{\Delta}_S \\
\mu \tilde{\Delta}_S \\
\mu \tilde{\Delta}_S \\
\mu \tilde{\Delta}_R
\end{bmatrix}
$$

where $\tilde{\Delta}_N$ and $\tilde{\Delta}_R$ are complex-valued weaknesses, which were generated from the real-valued weaknesses $\Delta_N$ and $\Delta_R$ in the following manner:

$$
\tilde{\Delta}_N = \Delta_N - i\Delta'_N , \quad \tilde{\Delta}_R = \Delta_R - i\Delta'_R.
$$

see also (MacBeth, 1999).

In equation (1), $\lambda$, $\mu$ are the Lame’s constants in the background rock, $M = \lambda + 2\mu$, $g = \mu(\lambda + 2\mu)/(\lambda + \mu)$, and $\xi$ is defined as $\xi = \lambda/(\lambda + 2\mu) = (V_s/V_p)^2$, where $V_s$ and $V_p$ are S-wave and P-wave velocities in the background rock. In equation (2), $\Delta_N$ and $\Delta_R$ are the normal and tangential weaknesses (Bakulin et al., 2000), which weaken the isotropic background rock due to the presence of fractures ($0 < \Delta_N < 1$, $0 < \Delta_R < 1$).
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We describe the anisotropy of attenuation by the matrix of the inverse quality factor $Q$, that is $\{Q^{-1}\}$, which can be expressed (Carcione, 2000) as

$$Q_{ij}^{-1} = \frac{\text{Im} \tilde{C}_{ij}}{\text{Re} C_{ij}},$$

where $\{\tilde{C}_{ij}\}$ is the complex-valued stiffness matrix given by equation (1). Then, the matrix $\{Q_{ij}^{-1}\}$ for the attenuative VTI medium can be written as

$$Q_{ij}^{-1} = \begin{bmatrix} L & K & J & 0 & 0 & 0 \\ K & L & J & J & 0 & 0 \\ J & J & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N & 0 & 0 \\ 0 & 0 & 0 & 0 & N & 0 \\ 0 & 0 & 0 & 0 & 0 & N \end{bmatrix},$$

where $\alpha$ is measured relatively the symmetry axis $z$.

Following Schoenberg and Douma (1988), we express the P-wave velocity $V_p(\alpha)$ as

$$V_{p}^{\perp}(\alpha) = V_p(1 - \Delta_{\alpha}[1 - 2g \sin^2 \alpha]^2 - \Delta_s \sin^2 2\alpha),$$

the SV-wave velocity $V_{sv}(\alpha)$ as

$$V_{sv}^{\perp}(\alpha) = V_s(1 - \Delta_{\alpha} - (g\Delta_s - \Delta_p) \sin^2 2\alpha),$$

and the SH-wave velocity $V_{q}(\alpha)$ as

$$V_{q}^{\perp}(\alpha) = V_q(1 - \Delta_{\alpha} \cos^2 \alpha),$$

where $\alpha$ is measured relatively the symmetry axis $z$.

We define attenuation (Carcione, 2000) as

$$Q^{-1} = \frac{\text{Im} \tilde{F}^{-1}(\alpha)}{\text{Re} \tilde{F}^{-1}(\alpha)},$$

where $\tilde{v}$ is complex-valued phase velocity, and then $\tilde{v}^2$ can be written as

$$\tilde{F}^{\perp}(\alpha) = \text{Re} \tilde{F}^{-1}(\alpha) + i \text{Im} \tilde{F}^{-1}(\alpha).$$

Within equations (5)-(7), we replace the weaknesses $\Delta_{\alpha}$ and $\Delta_{\alpha}$ with the complex-valued weaknesses $\tilde{\Delta}_{\alpha}$ and $\tilde{\Delta}_{\alpha}$ following equation (2). So we get complex-valued $\tilde{v}_{p}^{\perp}(\alpha)$, $\tilde{v}_{sv}^{\perp}(\alpha)$ and $\tilde{v}_{q}^{\perp}(\alpha)$. Then, we separate the imaginary and real parts, $\text{Im} \tilde{F}^{-1}(\alpha)$ and $\text{Re} \tilde{F}^{-1}(\alpha)$, and derive the attenuation $Q^{-1}(\alpha)$ following equation (8). By this manner we get the P-, SV-, and SH-wave attenuation as

$$Q_{p}^{\perp}(\alpha) = \frac{\Delta_{\alpha}[1 - 2g \sin^2 \alpha]^2 + \Delta_{\alpha} g \sin^2 2\alpha}{1 - \Delta_{\alpha}[1 - 2g \sin^2 \alpha]^2 - \Delta_s \sin^2 2\alpha},$$

$$Q_{sv}^{\perp}(\alpha) = \frac{\Delta_{\alpha} + (g\Delta_s - \Delta_p) \sin^2 2\alpha}{1 - \Delta_{\alpha} - (g\Delta_s - \Delta_p) \sin^2 2\alpha},$$

$$Q_{q}^{\perp}(\alpha) = \frac{\Delta_{\alpha} \cos^2 \alpha}{1 - \Delta_{\alpha} \cos^2 \alpha}.$$

Experiment versus theory

The model is constructed from rectangular plexiglas plates, each 1 mm thick, which were held on together by compressing uniaxial pressure 2 MPa. There is periodic variability of plate thickness (from 1.00 to 1.03 mm) with the spatial period of 10 mm. It is assumed that attenuation is due to linear slip interfaces between the plates. The ultrasonic experiment was performed for two models – with oil saturation and without any saturation (“dry model”).

The experimental data on the P-, SV- and SH-wave velocity and attenuation is presented in Figure 1 (marked by symbols), for the dry model (a, c) and the oil-saturated model (b, d). The angle $\alpha$ is measured relatively the symmetry axis $z$. Note that only the restricted $\alpha$-angle range – from 45° to 90° – is available in the experiment (see details in Gik & Bobrov (1996)).

To describe the shear-wave attenuation $Q_{s}^{\perp}(\alpha)$ and $Q_{q}^{\perp}(\alpha)$, observed in the experiment, we have to introduce the isotropic attenuation $Q_{iso}^{\perp}(\alpha)$. Then, equations (10) and (11) for $Q_{s}^{\perp}(\alpha)$ and $Q_{q}^{\perp}(\alpha)$ should be corrected as

$$Q_{s}^{\perp}(\alpha) \rightarrow Q_{s}^{\perp}(\alpha) + (Q_{iso}^{\perp}(\alpha)), \quad (12)$$

$$Q_{q}^{\perp}(\alpha) \rightarrow Q_{q}^{\perp}(\alpha) + (Q_{iso}^{\perp}(\alpha)), \quad (13)$$

where $(Q_{iso}^{\perp}(\alpha)) = 0.048$ in the oil-saturated model, and $(Q_{iso}^{\perp}(\alpha)) = 0.031$ in the dry model.

### Table 1. Model-input parameters for velocity

<table>
<thead>
<tr>
<th>$V_{L_\perp}$</th>
<th>$V_{L_\perp}$</th>
<th>$\epsilon$</th>
<th>$\delta$</th>
<th>$r$</th>
<th>$\Delta_{\alpha}$</th>
<th>$\Delta_{\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>963</td>
<td>2547</td>
<td>0.04</td>
<td>-0.21</td>
<td>0.47</td>
<td>0.48</td>
<td>0.10</td>
</tr>
<tr>
<td>Dry</td>
<td>879</td>
<td>1285</td>
<td>1.26</td>
<td>0.43</td>
<td>0.66</td>
<td>0.57</td>
</tr>
<tr>
<td>Oil-saturated</td>
<td>Dry</td>
<td>2680</td>
<td>1340</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2. Model-input parameters for attenuation

<table>
<thead>
<tr>
<th>$Q_{s}^{\perp}$</th>
<th>$Q_{s}^{\perp}$</th>
<th>$Q_{q}^{\perp}$</th>
<th>$Q_{q}^{\perp}$</th>
<th>$Q_{iso}^{\perp}$</th>
<th>$Q_{iso}^{\perp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>0.179</td>
<td>-0.77</td>
<td>-0.25</td>
<td>-0.68</td>
<td>0.053</td>
</tr>
<tr>
<td>Dry</td>
<td>0.526</td>
<td>-0.93</td>
<td>-1.79</td>
<td>-0.42</td>
<td>0.010</td>
</tr>
<tr>
<td>Oil</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.161</td>
</tr>
</tbody>
</table>

We found out certain theory-predicted interrelationships between the P- and S-wave velocities and attenuations in terms of the weaknesses $\Delta_{\alpha}$, $\Delta_{\alpha}$, $\Delta_{\alpha}$, $\Delta_{\alpha}$, that enabled us
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...to get the corrected (mutually compatible) values of the P- and S-wave symmetry-axis velocities and attenuations from the experimental data. (Because of the lack of data at the angles $\alpha$ from 0° to 45°, the straightforward estimation of the symmetry-axis velocities was unavailable.) Following the theory, we have substantially corrected the velocities, especially the SH-wave velocity $V_{\text{SH}}(\alpha)$ in the case of the dry-crack model; that is why it occurred to be not well-suited to the experimental data, as it is shown in Figure 1 (a), for the dry model. The same takes place in the case of the attenuation curves shown in Figure 1 (d), for the oil-saturated models.

Thomsen-style parameters for attenuation

Following Zhu & Tsvankin (2006), we write Thomsen-style anisotropy parameters for attenuation as

$$
\varepsilon_\parallel = \frac{Q_{\parallel}^2 - Q_{\perp}^2}{Q_{\parallel}^2}, \quad \gamma_\parallel = \frac{Q_{\parallel}^2 - Q_{\perp}^2}{Q_{\parallel}^2}, \quad \delta_\parallel = \frac{0.5 d^2 \delta_\parallel}{Q_{\parallel}^2} \bigg|_{\alpha = 0},$

where the symbol $\perp$ marks the direction of wave propagation normal to fracture planes (that is the symmetry-axis direction), and the symbol $\parallel$ marks the direction parallel to fracture planes (that is the isotropy-plane direction). In terms of the matrix \{ $Q_\parallel$, $Q_\perp$, $Q_{1\parallel}$, $Q_{1\perp}$, $Q_{2\parallel}$, $Q_{2\perp}$ \}, $Q_{1\parallel} = Q_{1\perp}$, $Q_{2\parallel} = Q_{2\perp}$, $Q_{1\parallel} = Q_{1\perp} = Q_{2\parallel}$, and $Q_{2\parallel} = Q_{2\perp}$.

For the P-wave attenuation, Thomsen-style approximation can be written (Zhu and Tsvankin, 2006) as

$$
\varepsilon_\parallel = \left( \frac{V_{\text{P}}}{V_{\text{P}}^\perp} \right)^2 - 1 \bigg|_{\alpha = 0},
$$

We found out that $\varepsilon_\parallel$ is independent of the attenuation:

$$
\varepsilon_\parallel = \left( \frac{V_{\text{P}}}{V_{\text{P}}^\perp} \right)^2 - 1 \bigg|_{\alpha = 0}.
$$

On the contrary, the parameter $\delta_\parallel$ does depend on the attenuation:
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\[ \delta_0 = 4 \left( \frac{Q_{\perp}}{Q_\parallel} - 1 \right) \frac{V_{\perp}^2}{V_\parallel^2} \]  

(16)

Following approximations of Zhu and Tsvankin (2006), we define the SH-wave attenuation as

\[ Q_{\perp}^{\parallel} (\alpha) = Q_\parallel^{\parallel} (1 + \gamma_{\perp} \sin^2 \alpha) \]  

(17)

where the \( \gamma_{\perp} \)-value is given in Table 2. The SV-wave attenuation is

\[ Q_{\perp}^{\parallel} (\alpha) = Q_\parallel^{\parallel} (1 + \gamma_{\perp} \sin^2 2\alpha) \]  

(18)

where the parameter \( \gamma_{\perp} \) was estimated as 5.15 for the dry model, and \( \gamma_{\perp} = -2.16 \) for the oil-saturated model. Following the sign of \( \gamma_{\perp} \) (plus or minus), the SV-wave attenuation curve shown in Figure 1(c) turns its pattern from the convex (for the dry model) to the concave (for the oil-saturated model), Figure 1(d). Unfortunately, to corroborate this, we have not got any reliable experimental data for SV-wave attenuation in the case of the dry model. Note that in equations (17) and (18), the \( Q_{\perp}^{\parallel} \)-value given in Table 2 should be corrected for the isotropic attenuation of oil-saturated and dry models. Actually, after oil saturation, the S-wave attenuation \( Q_{\perp}^{\parallel} \) increases:

\[ \frac{Q_{\perp}^{\parallel}}{Q_\parallel^{\parallel}} = 4.6 \]

but at the same time, the P-wave attenuation \( Q_{\perp}^{\parallel} \) decreases:

\[ \frac{Q_{\perp}^{\parallel}}{Q_\parallel^{\parallel}} = 0.34 \]

This resulted in the essential contrast in \( \delta_0 \)-values for the oil-saturated and dry models, that is

\[ \delta_0^{\text{IS}} = 7 \delta_0^{\text{OIL}} \]  

(19)

Actually, it follows from equation (16) that \( \delta_0 \) can be defined by the relative difference between S- and P-wave attenuations \( Q_{\perp}^{\parallel} \) and \( Q_\parallel^{\parallel} \):

\[ \delta_0 = \left( \frac{Q_{\perp}^{\parallel}}{Q_\parallel^{\parallel}} \right) - 1 \]

if assumed \( V_{\perp}^{\parallel}/V_\parallel^{\parallel} = 0.5 \) in equation (16).

Thus, the essential difference in \( \delta_0 \)-values for the oil-saturated and dry models is caused by the great change in the \( Q_{\perp}^{\parallel}/Q_\parallel^{\parallel} \)-value for the oil-saturated and dry models,

\[ \left( \frac{Q_{\perp}^{\parallel}}{Q_\parallel^{\parallel}} \right)^{\text{OIL}} = 14 \left( \frac{Q_{\perp}^{\parallel}}{Q_\parallel^{\parallel}} \right)^{\text{IS}} \]

Thus, we found out that the symmetry-axis \( Q_{\perp}^{\parallel}/Q_\parallel^{\parallel} \)-ratio is much larger in a wet-crack medium than in a dry-crack medium, \( (Q_{\perp}^{\parallel}/Q_\parallel^{\parallel})^{\text{OIL}} > (Q_{\perp}^{\parallel}/Q_\parallel^{\parallel})^{\text{IS}} \). This is in accordance to laboratory measurements at ultrasonic frequencies of P- and S-wave quality factors (Tokšić et al., 1979), as well as attenuation data from sonic logs (Klimentos, 1995) that show \( (Q_{\perp}^{\parallel}/Q_\parallel^{\parallel})^{\text{OIL}} > (Q_{\perp}^{\parallel}/Q_\parallel^{\parallel})^{\text{IS}} \). (Note that \( Q_{\perp}/Q_\parallel = Q_{\perp}^{\parallel}/Q_\parallel^{\parallel} \))

In particular, they have observed \( Q_{\perp}/Q_\parallel > 1 \) for wet rocks (with water or brine) and \( Q_{\perp}/Q_\parallel < 1 \) for dry (or gas-saturated) rocks.

**Conclusions**

The experimental data was used as an illustration material for the theory application. We managed with the lack of data at the angles from 0° to 45° to the symmetry axis, this situation is similar to that which takes place in the case of seismic reflection data, in frames of HTI model.

The behavior of P-, SV- and SH-wave attenuation as a function of the wave-propagation direction is similar in common to the well known directional dependences for the velocities in TI medium. The attenuation curves are alike the velocity curves according to the principle “the maximum of velocity curve corresponds to the minimum of attenuation curve, and vice versa”.

Attenuation anisotropy is much stronger than velocity anisotropy. For example, in the oil-saturated model, the P-wave velocity anisotropy is 4%, but the anisotropy of attenuation is much greater, 115%. In real seismic data, the P-wave velocity anisotropy caused by liquid-saturated cracks is very small and hardly can be used for seismic exploration. However, the P-wave attenuation anisotropy is rather strong and can be used in fracture characterization, for estimation of fracture direction in reservoir rocks from azimuthally varying Q. Examples are in Vasconselos & Jenner (2005), Maulztsch et al. (2005), Varela et al. (2006) and Chichinina et al. (2005, 2006a, 2006b).

We found out that the ratio of the symmetry-axis P- and S-wave attenuations, \( Q_{\perp}/Q_\parallel \), and the parameter \( \delta_0 \) are interlinked and strongly dependent of the fluid type in cracks, that is confirmed by our ultrasonic experiment. Therefore, we believe that analysis of Q-anisotropy may get potential use for seismic exploration, for example the analysis of Q versus offset (QVO) in seismic reflection data, in frames of HTI model. An estimate of \( Q_{\perp}/Q_\parallel \) (\( Q_{\perp}/Q_\parallel < 1 \) or \( Q_{\perp}/Q_\parallel > 1 \)) may provide an additional information for fracture-reservoir characterization (Tokšić et al. (1979), Klimentos (1995), Winkler (1979), Tiwari and McMechan (2003), and Batzle et al. (2005)).
EDITED REFERENCES
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