Generalization of Schoenberg’s Linear Slip model to attenuative media: physical modeling versus theory

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Summary

We propose a mathematical formalism of introducing attenuation in Schoenberg’s Linear Slip model. This formalism consists in replacing the real-valued weaknesses, normal and tangential, entering in the stiffness matrix by the complex-valued quantities. To test the validity of this procedure, an ultrasonic experiment was performed with the use of a plate-stack model made from plexiglass plates. We measured velocities and attenuations of P-, SH- and SV-wave versus the angle between the ray and the symmetry axis of TI model for air-filled and oil-saturated state of the model under uniaxial pressure of 2 and 4 MPa. The data on velocities and attenuations were fitted by the derived theoretical functions. We have estimated the values of the imaginary and real parts of the complex-valued weaknesses. The real parts of the weaknesses are threefold for the air-filled model in comparison with the oil-saturated one. The imaginary parts of the weaknesses, responsible for attenuation, are one order of magnitude less than the values of the real parts of the weaknesses. Both P-wave anisotropies, the velocity anisotropy and the attenuation anisotropy, are greater in the air-filled model than in the oil-saturated one. Besides, in the air-filled model, the symmetry-axis attenuation of the P-wave is much greater than the S-wave symmetry-axis attenuation, whereas in the oil-saturated model these attenuations are similar.

Introduction

In this paper we study the attenuation anisotropy in a linear slip model. The linear slip model in various modifications is proposed by Klem-Musatov (Klem-Musatov et al., 1973; Aizenberg et al., 1974), Schoenberg (1980, 1983), Kitsunezaki (1983), Pyrak-Nolte et al. (1990a, 1990b), and Gu et al. (1996). The linear slip model describes elastic wave propagation in media with imperfectly bonded interfaces. These interfaces may be due to bedding planes, joints, fractures, populations of coplanar cracks, thin soft layers, etc. Displacements across such interfaces are not continuous, i.e., the contacts between thin layers are unwelded, whereas the stress (that is the traction across the interface) should be continuous.

The Linear Slip (LS) theory, advanced by Michael Schoenberg for a single interface in 1980 and for a periodically stratified elastic medium in the long wavelength limit in 1983, is developed by him with coauthors during the years which followed (Schoenberg and Douma, 1988; Hood and Schoenberg, 1989; Hsu and Schoenberg, 1993; Schoenberg and Sayers, 1995; Schoenberg and Nakagawa, 2006, etc.). A number of problems related primarily to detection of vertical fracturing have been solved. The validity of the linear slip theory is proved experimentally by means of measuring compressional (P) and shear (S) wave velocities in a block composed of lucite plates pressed together to simulate a fractured medium (Hsu and Schoenberg, 1993).

The possibility for extension of the linear slip theory to viscoelastic (or attenuative) media is mentioned by Schoenberg already in his first publication (1980) on the LS model. He writes that the displacement discontinuity dependence on traction vector “may be complex and frequency dependent corresponding to a viscoelastic spring condition”. Further he refines on the thought: “... As real elastic parameters may be generalized to complex frequency dependent viscoelastic parameters via the harmonic elastic–viscoelastic analogy (Bland, 1960), so may the slip boundary compliances be generalized allowing the modeling of a linear viscoelastic slip interface”. Moreover, in solving the problem on reflection – transmission of plane SH waves at a linear slip interface, Schoenberg, in particular, considers the case of a pure viscous slip interface. Besides, Schoenberg and Muir (1989) consider the possibility for the introduction of “group elements from viscoelastic layers” as a direct extension of their theory developed for the problem of averaging elastic modules of thin-layered media with anisotropic layers by means of matrix algebra and group theory formalism.


Chichinina et al. (2006a, 2006b) pioneered in introducing the attenuation in the linear slip model. The real-valued weaknesses entering in the stiffness matrix of LS HTI medium are substituted by their complex-valued analogs, and the expression for P-wave attenuation is derived. The azimuthal variations of P-wave attenuation are studied for the Hudson’s model (1981) containing penny-shaped aligned cracks with gas and oil infill. In the long
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wavelength limit, this model and the LS model are equivalent (e.g., Bakulin et al., 2000). The imaginary parts of the weaknesses are interpreted in the terms of the equant-porosity Hudson’s model, in which attenuation is caused by the local fluid flows between aligned cracks and randomly distributed pores (Point et al., 2000).

Now we continue the study of attenuation anisotropy (or the Q-anisotropy) in the LS model started in (Chichinina et al., 2007a, 2007b, 2008) with developing the theoretical formalism and testing its validity on the data of the laboratory ultrasonic experiment. Attention is given to justification of the replacement of the real-valued weaknesses by complex-valued and possible attenuation mechanisms.

Attenuation anisotropy of P- and S-waves in rocks is studied in laboratory ultrasonic experiments by Yin and Nur (1992), Best (1994), Kern et al. (1997), Kim et al. (1983), Domneateau et al. (2002), Jakobsen and Johansen (1999, 2000), Prasad and Nur (2003), Shi and Deng (2005), and Best et al. (2007). Besides ultrasonic experiments, there are several field experiments on fracture-direction estimation from azimuthally varying P-wave attenuation in surface seismic reflection data and VSPs (Clark et al., 2005; Varela et al., 2006; Best et al., 2007). Besides ultrasonic experiments, there are several field experiments on fracture-direction estimation from azimuthally varying P-wave attenuation in surface seismic reflection data and VSPs (Clark et al., 2005; Varela et al., 2006; Best et al., 2007). Beside ultrasonic experiments, there are several field experiments on fracture-direction estimation from azimuthally varying P-wave attenuation in surface seismic reflection data and VSPs (Clark et al., 2005; Varela et al., 2006; Best et al., 2007).

In laboratory experiments on anisotropy, it is preferable to simulate an ideal transversely isotropic medium using synthetic specimens formed by thin micro-layers, or models with embedded parallel penny-shaped cracks, e.g. Rathore et al. (1994), Sothcott et al. (2007), Wei et al. (2008), Wei and Di (2008). As for the model manufacturing, a plate-stack model can be considered as the simplest model of TI medium (e.g., Hsu and Schoenberg, 1993; Zhu et al., 2007a). Such a plate-stack model suits ideally for the simulation of Schoenberg’s linear slip model.

The experiment of Hsu and Schoenberg (1993) with the plate-stack model made from lucite plates validates the stiffness matrix of Hsu and Schoenberg (1993) by introducing the complex-valued weaknesses instead of the real-valued weaknesses:

\[
\tilde{c} = \begin{bmatrix}
M(\lambda - \xi \Delta) & L(\lambda - \xi \Delta) & R(\lambda - \xi \Delta) & 0 & 0 & 0 \\
L(\lambda - \xi \Delta) & M(\lambda - \xi \Delta) & R(\lambda - \xi \Delta) & 0 & 0 & 0 \\
R(\lambda - \xi \Delta) & R(\lambda - \xi \Delta) & M(\lambda - \xi \Delta) & 0 & 0 & 0 \\
0 & 0 & 0 & \mu(\lambda - \Delta) & 0 & 0 \\
0 & 0 & 0 & 0 & \mu(\lambda - \Delta) & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}
\]

(1)

Here \(\lambda\) and \(\mu\) are the Lamé constants of the isotropic background (\(M = \lambda + 2\mu, \quad \xi = \lambda/M\)), and \(\Delta_N, \Delta_T\) are the complex-valued weaknesses, normal \(\Delta_N = \Delta_N - i\Delta_N'\) and tangential \(\Delta_T = \Delta_T - i\Delta_T'\), where \(0 < \Delta_N < \Delta_N < 1, 0 < \Delta_T < \Delta_T < 1\). In consequence, the complex-valued Christoffel tensor \(\tilde{c}_{ij} = n_{ijkl}n_{ji}n_{kl}\) (where \(n\) is density and \(\beta\) is the wave-normal vector) yields three complex-valued eigenvalues, which are the squared phase velocities, \(\beta_{P, SV, SH}^2\) where \(P, SV, SH\):

\[
V_T(\alpha) = V_S[1 - (\Delta_N - i\Delta_N') - (\Delta_T - i\Delta_T') - (\Delta_N - i\Delta_N')(\Delta_T - i\Delta_T')]^{1/2}
\]

\[
V_S(\alpha) = V_S[1 - (\Delta_N - i\Delta_N') - (\Delta_T - i\Delta_T')]^{1/2}
\]

\[
V_N(\alpha) = V_S[1 - (\Delta_N - i\Delta_N')]^{1/2}
\]

where \(g = (V_S/V_T)^2\), \(V_S\) and \(V_N\) are P- and S-wave velocities in the background, and \(\alpha\) is the wave-propagation angle measured relatively the symmetry axis of the VTI medium, which is the axis \(z\). The sign “\(\approx\)” is due to neglecting \(\alpha(\Delta')\) terms.

The attenuation, which is the inverse quality factor \(Q\), can be expressed (e.g., Carcione, 2001) through the complex-valued squared phase velocity \(\beta^2\) as \(Q^{-1} = \text{Im}(\beta^2)/\text{Re}(\beta^2)\). Then using equations 2, we get the following expressions for the P-, SH- and SV-wave attenuations:

\[
\begin{align*}
Q_P^\perp(\alpha) &= \frac{\Delta_N'[1 - 2g \sin^2 \alpha] + \Delta_N g \sin^2 2\alpha}{1 - \Delta_N'[1 - 2g \sin^2 \alpha] - \Delta_N g \sin^2 2\alpha}, \\
Q_S^\parallel(\alpha) &= \frac{\Delta_N' + (g\Delta_N - \Delta_N') \sin^2 2\alpha}{1 - \Delta_N' - (g\Delta_N - \Delta_N') \sin^2 2\alpha}, \\
Q_S^\perp(\alpha) &= \frac{\Delta_N \cos^2 2\alpha}{(1 - \Delta_N) \cos^2 2\alpha}.
\end{align*}
\]

(3)

(4)

(5)

Expansion of equation 3 for \(Q_P^\perp(\alpha)\) in Taylor series in \(\sin^2 \alpha\) in the vicinity of \(\alpha = 0\) is
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\[ Q_p^{-1}(\alpha) = A + Bx + Cx^2, \]
\[ x = \sin^2 \alpha, \quad A = Q_p^0(0) = Q_p^0, \quad B = d \frac{d Q_p}{d(\alpha)} \bigg|_{\alpha=0}, \]
\[ C = 0.5d^2 \frac{d^2 Q_p}{d(\alpha)^2} \bigg|_{\alpha=0}, \]

where \( A, B, \) and \( C \) are derived as functions of \( \Delta_\nu, \Delta_f, \Delta'_\nu, \Delta'_f, \) and \( g \) (Chichinina et al., 2007a, 2007b, 2008).

By the same manner, we obtain the approximations for the SV- and SH-wave attenuations:

\[ Q_s^0(\alpha) \approx A_x + B_{xy} \sin^2 2\alpha, \]
\[ Q_{sh}^0(\alpha) \approx A_x + B_{shy} \sin^2 \alpha. \]

Physical modeling

The plate-stack model (Gik and Bobrov, 1996) consists of rectangular plexiglass plates, each 1 mm thick, 7.2 x 25 cm², the total height of the model is 25 cm (Figure 1). The model is loaded by the uniaxial pressure (\( P = 2 \) MPa; 4 MPa). Each plate is characterized by a gradual change of its thickness by a value of about 0.01-0.03 mm, because of technological factors. The plate-thickness-distribution function in the plate plane is a random function with a period of about 10 mm. The compression pressure applied to the plate-stack model leads to appearing of the contact areas between the plate surfaces which form the contact layers; they may be called as “soft layers” following the terminology of Schoenberg and Nakagawa (2006). The average thickness of these soft layers is \( \sim 0.001 \) mm at the loading pressure of 2 MPa. Such peculiarities of the inter-plate contacts give an insight into the possible mechanism of attenuation in the model under consideration. The attenuation may be due to scattering on the plate-surface undulations or, in a more general treatment, may be due to increased attenuation at the thin “soft layers”. It is worth noting that in the lucite plate-stack model of Hsu and Schoenberg (1993), the attenuation may be considered also due to the “soft layers”. The surfaces of lucite plates were roughened by sand blasting to attain better results than with smooth plates. In this case, the details of the attenuation mechanism may be also explained in terms of the friction between the asperities at the two contacting rough surfaces. Such a model is described by Walsh and Grosenbaugh (1979). In the case of saturation, the most probable cause of attenuation in both models – of Gik and Bobrov (1996) and Hsu and Schoenberg (1993) – is in fluid flows between the fractures in these thin “soft layers”.

The measurements of velocity and attenuation of P-, SV-, and SH-waves are performed for various directions of the source-to-receiver line (SR) given by the angle \( \alpha \). By the same manner, we obtain the approximations for the SV- and SH-wave attenuations:

\[ Q_s^0(\alpha) \approx A_x + B_{xy} \sin^2 2\alpha, \]
\[ Q_{sh}^0(\alpha) \approx A_x + B_{shy} \sin^2 \alpha. \]

The measurements of velocity and attenuation of P-, SV-, and SH-waves are performed for various directions of the source-to-receiver line (SR) given by the angle \( \alpha \) relatively the symmetry axis \( z \) (Figure 1). The rays SR are going in the shaded plane (xz). The horizontal ray makes the angle \( \alpha = 90^\circ \) with z-axis. The symmetry-axis angle \( \alpha = 0^\circ \) is inaccessible because the upper and lower model sides are covered with steel plates fastened together by two screws located outside the model to produce the uniaxial pressure \( P \). Thus, the available \( \alpha \) interval is \( (25^\circ, 90^\circ) \).

The experiment was performed for the two states of the model: oil-saturated and air-filled (dry). In the first case, the plates were immered into motor oil before the assembly of the model. The emitters and receivers used in the experiment are 3-component contact piezoelectric transducers designed and fabricated by Bobrov et al. (1984). The diameter of the transducer is 10 mm; the central frequency \( f = 100 \) kHz. The P-wave velocity in plexiglass, the plates’ material, \( V_p \approx 2800 \) m/s, then the wave length \( \lambda_p = V_p / f = 28 \) mm. For field parameters in real seismic data, for example, at the frequency \( f_{\text{field}} = 28 \) Hz, this means the wavelength \( \lambda_{\text{field}} = V_p / f_{\text{field}} = 100 \) m. In the ultrasonic experiment, the fracture spacing (the plate thickness) \( h = 1 \) mm, which gives \( h / \lambda_p \approx 0.04, \) and the fracture opening (the spacing between the plates) \( \Delta h \approx 0.001 \) mm. Thus, the constraint for the thin-layered LS model \( \lambda >> h >> \Delta h \) (e.g., Bakulin

![Figure 1](image1.png)

**Figure 1.** The model view and the scheme of the experiment. Shown are the location of the sources (S) and receivers (R), the angle \( \alpha \) between the rays SR and z-axis, and the direction of the uniaxial pressure \( (P = 2 \) MPa and \( P = 4 \) MPa).

![Figure 2](image2.png)

**Figure 2.** P-, SV-, and SH-wave velocity \( V \) and attenuation \( Q^{-1} \) versus the angle \( \alpha \) (shown in Figure 1) for the oil-saturated (Oil) and air-filled (Dry) models under the pressure of 2 MPa.
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et al., 2000) is fulfilled. For S-wave, \( V_S = 1300 \text{ m/s} \), then \( \lambda_S = 13 \text{ mm}, h \approx 0.08\lambda_S \), and the above inequalities also are satisfied. By comparison, in the model of Hsu and Schoenberg (1993), also \( h/\lambda_p = 0.04, h/\lambda_S \approx 0.075 \) (at the central frequency \( f = 150 \text{ kHz} \)).

For Q-estimation, we use the technique based on the Spectral Ratio method (e.g., Toksöz et al., 1979; see also Chichinoa et al., 2006b; Zhu et al., 2007a). For the reference attenuation, we made use of the attenuation estimated in the plexiglass unfractured block of the same size as the plate-stack model. [Hsu and Schoenberg (1993) did the same using the lucite unfractured block.] The technique of Q-estimation in the solid plexiglass block consisted in comparison of the spectra of the transmitted wave and the reflected wave.

The measurement errors for velocity do not exceed 2 %, and for attenuation 5 % (Gik and Bobrov, 1996). The accuracy of the Q-measurements depends also on other assumptions. For example, the radiation pattern of the source and the geometrical spreading are taken to be frequency independent in the frequency band used in the spectral-ratio method; the details on justification of these assumptions can be found in Zhu et al. (2007a). However, the misfits between the data and the corresponding theoretical “exact functions” (shown in Figure 2) are much greater than the measurement accuracy. We assume that the cause of this misfit may be in the inhomogeneity of the model induced by uneven distribution of stress field inside the model because of the imperfect technical means of the pressure loading by two screws.

Experimental results

The experimental data for the P-, SV-, and SH-wave velocity and attenuation are presented in Figure 2 (the loading pressure \( P = 2 \text{ MPa} \)). Most data for SV-wave are absent, because of the fuzzy arrivals due to the wave interference. The experimental values of attenuations were at first fitted by the approximate functions \( Q^{-1}(\alpha) \), equations 6–7, and then by “exact” equations 2, 3–5. The estimated values of the real weaknesses are \( \Delta_\alpha \approx \Delta_\beta = 0.5–0.6 \) in the dry model, whereas in the oil-saturated one \( \Delta_\alpha \approx \Delta_\beta = 0.15–0.25 \). These results are comprehensible, since the real weaknesses have the distinct physical meaning – they weaken the material. In the experiment, they are threefold for the dry model in comparison with oil-saturated. It is of interest to compare the above estimates with those for analogous plate-stack model of Hsu and Schoenberg (1993). In their air-filled model, \( \Delta_\alpha = 0.39 \), \( \Delta_\beta = 0.19 \); in the honey-saturated model, the values of both compliances, \( \Delta_\alpha \) and \( \Delta_\beta \), do not exceed 0.05. These values are in rather good agreement with our estimates.

The estimated values of the imaginary parts of the normal and tangential weaknesses, \( \Delta'\alpha \) and \( \Delta'I \), are \( \sim 0.01–0.1 \) for both models, air-filled and oil-saturated. These values are also comprehensible: \( \Delta'\alpha \) and \( \Delta'I \) are responsible for attenuation, and their values are one order of magnitude less than \( \Delta_\alpha \) and \( \Delta_\beta \).

The data of the experiment show that the attenuation anisotropy of P- and S-waves is much greater than the velocity anisotropy. This result is in line with the results of other authors, e.g. the most recent experiment of Zhu et al. (2007a) performed for the P-wave attenuation anisotropy in a composite plate-stack sample made from phenolic material. It is clearly seen (Figure 2) that the behavior of the attenuation function is reciprocal to the velocity function, in accordance with the principle “the maximum of velocity curve corresponds to the minimum of attenuation curve, and vice versa”. The observed attenuation anisotropy exhibits the same VTI symmetry as the velocity anisotropy.

Among the main experimental results, there is also the fact that both P-wave anisotropies, the velocity anisotropy and the attenuation anisotropy, are greater in the dry model than in the oil-saturated one. In the dry model, the symmetry-axis attenuation of the P-wave, \( Q_{\perp}^{\perp} \), is much greater than the S-wave symmetry-axis attenuation \( Q_{\perp}^{\perp} \), whereas in the case of the oil-saturated model these attenuations are similar.

Conclusions

The validity of the proposed mathematical formalism of introducing attenuation in Schoenberg’s Linear Slip model is proved by the laboratory ultrasonic experiment on plexiglass plate-stack model. The experimental values of the velocities and attenuations fitted by the derived theoretical functions occur generally to be in agreement. However, the matching is far from ideal. We believe that the major cause of the observed imbalance in matching of the theory-predicted curves to the data is in the inhomogeneity of the model arisen as a consequence of imperfectness of the technique used for pressing the plates together to form the plate-stack model. This unexpected situation created difficulties in the data inversion, under the assumption of the model homogeneity, for the complex-valued weaknesses, in particular in joint inversion of P- and S-wave data. Nevertheless, this circumstance may be considered as useful since it reminds that the real stress field in earth subsurface also is not homogeneous. Hence, the future work should include development of new models accounting for the complicated anisotropic state of the real media due to a non-uniform distribution of stresses in the subsurface.
EDITED REFERENCES
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