AVOA algorithm for fracture characterization
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Summary
Nowadays the analysis of azimuthal variation in reflection coefficients or AVOA analysis (Amplitude Versus Offset and Azimuth) is widely applied for detection and mapping highly fractured zones with azimuthally-oriented vertical cracks. Based on Rüger’s equations for reflection coefficients in HTI medium, AVOA technique gives, in general case, two symmetry directions of HTI medium without any formal indication which of these two orthogonal directions points to the symmetry axis and which to the fracture strike. Therefore some additional information is required besides reflection coefficients to resolve the ambiguity in fracture-direction estimation. We develop a new computational algorithm for P-wave AVOA analysis (improved in comparison with existing technique) which resolves the ambiguity using only reflection coefficients. The algorithm makes use of the third term in Rüger’s approximation for determining symmetry-axis and fracture-strike azimuths uniquely. We test the algorithm in synthetic and field data.

Introduction
The methodolgy of AVOA analysis is based on the concept of azimuthal anisotropy caused for the most part by parallel vertical fractures. It leads to the azimuthal anisotropy of amplitudes, in particular, to azimuthal variation in reflection coefficients. Fractured reservoir is represented by the model of a transversally isotropic medium with horizontal symmetry axis (HTI medium). The PP-wave reflection coefficient \( R \) at the interface between weakly anisotropic HTI media (or between isotropic and HTI medium) is defined by the approximate formula (Rüger, 1998)

\[
R(\theta, \phi) = A + B(\phi) \sin^2 \theta + C(\phi) \sin^2 \theta \tan^2 \theta ,
\]

where \( \theta \) is the incidence angle, \( \phi \) is the source-receiver-line azimuth with respect to the coordinate axis \( x \), oriented along symmetry axis of HTI layer. The term \( A \) is the normal-incidence reflection coefficient,

\[
A = \Delta Z / (\Delta Z) ,
\]

where \( Z = \rho v^2 \) is the vertical P-wave impedance; the symbol \( \Delta \) denotes a difference between the parameters below and above the reflecting boundary and the bar \( \ldots \) is for the mean value of the parameters. The coefficient \( B(\phi) \) is so-called AVO gradient, which can be written as

\[
B(\phi) = B_{\phi\phi} + B_{\phi\psi} \cos^2(\phi - \phi_0) ,
\]

The symmetry axis forms an angle \( \phi_0 \) with \( x \)-axis. The term \( B_{\phi\phi} \) is the AVO-gradient isotropic part (equal to the AVO gradient for isotropic media), and \( B_{\phi\psi} \) is in the anisotropic part of AVO gradient,

\[
B_{\phi\psi} = \frac{1}{2} \Delta \delta^{(\phi)} - (2 F_1^\phi \Delta \rho_{\phi\phi}^\psi) ,
\]

where \( F_1^\phi \) and \( F_2^\phi \) are the vertical velocities (or the velocities in the isotropy plane) of P wave and S wave, respectively. Following the terminology of Rüger (1998) and Tsvankin (1997), the Thomsen-style anisotropy parameters \( \varepsilon^{\psi} \), \( \delta^{(\phi\psi)} \) and \( \rho^{(\phi\psi)} \) are used to describe anisotropy of HTI media; these parameters are always negative for the effective fractured medium HTI. The difference in the anisotropy parameters across the boundary is written as \( \Delta \delta = \delta^{(\phi\psi)} - \delta^{(\phi\psi)} \), where the subscripts \( \psi \) and \( \phi \) refer to the upper and lower medium, respectively.

The last term in equation (1), that is the term with the coefficient \( C(\phi) \), gives a noticeable contribution to the reflection coefficient value only for far offsets (large incidence angles). The coefficient \( C(\phi) \) can be written as

\[
2C(\phi) = \Delta \rho \cos^2(\phi - \phi_0) + \Delta \delta \sin^2(\phi - \phi_0) + \Delta \psi \cos^2(\phi - \phi_0) ,
\]

where \( \Delta \rho = \rho_1^\phi / \rho_2^\phi \) and

\[
\Delta \psi = \varepsilon^{\psi} / \varepsilon^{\psi} - \varepsilon^{\psi} / \varepsilon^{\psi} .
\]

The main problem is to estimate the symmetry-axis angle \( \phi_0 \). The algorithm is based on the equations (1)-(6); here we apply it to synthetic and field data. Note that in equation (1) is intended for the calculation of reflection coefficient, while in real data, the computational method deals with amplitudes, but not with the reflection coefficients. Therefore, for application, it is necessary to correct amplitudes for the geometrical spreading. Also, a proper definition of the amplitudes from wavelet samples is required because the amplitude-value estimation is very sensitive to noise.

The algorithm
The algorithm may be divided into three main steps. The first step consists in estimating of the coefficients \( A, B \) and \( C \) in equation (1). The second step is in defining two principle symmetry direction of HTI medium using the formula (3) for the AVO gradient \( B \). The third step is in analyzing the coefficient \( C \), equation (5), to distinguish the symmetry axis direction from the fracture strike.

The first step of the algorithm is the estimation of \( B(\phi) \) coefficient for each individual azimuth-sectored CMP gather. The coefficients \( A, B \) and \( C \) in equation (1) are determined by the least-squares method. For each source-to-receiver line with a number \( j \) (\( j = 1, \ldots, n \)), the functional
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\[ F = \sum |D_i - R_i| \]  

(7)

should be minimized. Here \( i \) is the number of offset (corresponding to the incidence angle \( \theta_i \)) in the \( j \)-th azimuth-sectored CMP gather, and \( R_i \) is the right-hand side of the formula (1), in which the value of the incidence angle \( \theta_i \) is substituted; \( D_i \) is the amplitude value estimated from real data for the incidence angle \( \theta_i \). The coefficients \( A, B \) and \( C \) are determined from the system of three equations:

\[ \frac{\partial F}{\partial A} = 0, \quad \frac{\partial F}{\partial B} = 0, \quad \frac{\partial F}{\partial C} = 0. \]  

(8)

Due to azimuthal anisotropy of the medium, AVO gradient \( B \) acquires different values \( B_1, B_2, \ldots, B_n \) for \( n \) azimuth-sectored CMP gather.

The second step of the algorithm consists in the estimation of the angle \( \phi_i \) from equation (3). For that, at least three AVO-gradient values are required, each of them estimated for its own source-to-receiver line azimuth (or sector). (For \( n = 3 \) the problem was solved by Mallik et al., 1998).

As \( A \) does not depend on azimuth, we should normalize AVO gradient \( B \) as \( B_j / A_j = B_j / A_j, \) (where \( A_j \) and \( B_j \) are calculated by the least-squares method for each azimuth sector \( j \)), and rewrite the formula (3) in the following manner

\[ B_j = a + b \cos[2(\phi_j - \phi_b)], \]  

(9)

where \( \phi_j \) is the mean azimuth of the \( j \)-th azimuth sector.

The values of \( B_j = B_j / A_j \) are already known (estimated at the first step), and the parameters \( a, b, \) and \( \phi_b \) are unknowns and can be obtained by the least-squares method by minimizing the functional

\[ f = \sum_{j=1}^n [a + b \cos(2\phi_j - \phi_b)] - B_j]^2. \]  

(10)

It yields the system of equations:

\[ \frac{\partial f}{\partial a} = 0, \quad \frac{\partial f}{\partial b} = 0, \quad \frac{\partial f}{\partial \phi_b} = 0. \]  

(11)

From equations (11), we derive an analytical trigonometric equation for \( \phi_i \) and find out that the solution \( \phi_b \) has a period of \( \pm\pi \), so that we can not be sure if \( \phi_b \) is the symmetry-axis azimuth or the orthogonal direction that is the fracture-strike. Also, from equations (9) one can infer two different solutions: one is \((a, b, \) and \( \phi_b) \) and the other is \((a, -b, \) and \( \phi_b \pm \pi) \), see also (Zhenheng et al., 2004). Both these points are the local minimum points, that is confirmed by calculation of the second partial derivatives of the functional \( f \).

How to distinguish fracture strike from the symmetry axis

Let's rewrite equation (5) as

\[ 2C_j = \Delta \alpha + \Delta \epsilon \cos^2(\phi_j - \phi_b) + \Delta \eta \sin^2(\phi_j - \phi_b) \cos^2(\phi_j - \phi_b), \]  

(12)

where \( j = 1, \ldots, n, \Delta \eta = \Delta \delta - \Delta \epsilon, \) and \( \Delta \epsilon \) is given by equation (6).

If substitute the value of \( \phi_b \pm \pi \) instead of \( \phi_b \) into equation (12), then the sign of its second term, \( \Delta \epsilon \cos^2(\phi_j - \phi_b) \), turns to the opposite sign, because equation (12) transforms to the following

\[ 2C_j = (\Delta \alpha + \Delta \epsilon) - \Delta \epsilon \cos^2(\phi_j - \phi_b) + \Delta \eta \sin^2(\phi_j - \phi_b) \cos^2(\phi_j - \phi_b). \]

Note that \( \Delta \epsilon \) should be negative for the upper reflecting boundary of the HTI layer (that is the case of HTI layer overlain by an isotropic overburden), and \( \Delta \epsilon > 0 \) for the lower boundary (i.e., for the interface HTI medium – isotropic medium). This follows from equation (6), with taking into account \( \epsilon(\prime) > 0 \) in isotropic medium and \( \epsilon(\prime) < 0 \) in HTI medium.

Thus, the sign of \( \Delta \epsilon \) is predefined for the model. If, for a given \( \phi_b \), the sign of \( \Delta \epsilon \) (calculated from equation (12)) does not satisfy the requirement on its sign, then it means that we deal with the case of fracture-strike direction, because the sign of the term \( \Delta \epsilon \cos^2(\phi_j - \phi_b) \) changes for this case. So, if the sign of \( \Delta \epsilon \) would be estimated, the ambiguity problem could be resolved.

We should normalize both sides of equation (12), again by \( A_j \) as that was done for equation (9), and beginning from now, assume \( C_j = C_j / A_j, \Delta \alpha = \Delta \alpha / A_j, \Delta \epsilon = \Delta \epsilon / A_j, \) and \( \Delta \eta = \Delta \eta / A_j. \)

Then, the value of new \( \Delta \epsilon \) can be estimated by minimizing the following functional \( f_i \), by the least-squares method:

\[ f_i = \sum_{j=1}^n [\Delta \alpha + \Delta \epsilon \cos^2(\phi_j - \phi_b) + \Delta \eta \sin^2(\phi_j - \phi_b) \cos^2(\phi_j - \phi_b) - 2C_j]^2. \]  

(13)

\[ \frac{\partial f_i}{\partial \Delta \alpha} = 0, \quad \frac{\partial f_i}{\partial \Delta \epsilon} = 0, \quad \frac{\partial f_i}{\partial \Delta \eta} = 0, \]  

where \( \phi_b \) is the known solution of the system (11).

The system of equations (13) is solved by analogy with the system of equations (7)-(8).

Keeping in mind the sign requirements for \( \Delta \epsilon \), we infer the following criterion for identification of the solution \( \phi_b \) as the symmetry-axis azimuth:

For upper reflecting boundary of the HTI layer

\[ \text{sign}(A) \Delta \epsilon < 0; \]  

(14)

for lower reflecting boundary of the HTI layer

\[ \text{sign}(A) \Delta \epsilon > 0. \]  

(15)

In the criterion, the value of \( \Delta \epsilon \) is assumed to be calculated from equation (13); and the term \( \text{sign}(A) \) is the sign of the normal-incidence reflection coefficient \( A \), that can be
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We apply the algorithm to synthetic data (Figure 1) and to field data (Figure 2). The synthetic data is represented by three CDP seismograms corresponding to three source-to-receiver-line azimuths 0°, 45° and 90°. These synthetic seismograms were generated by the ray tracing method (Obolentseva and Grechka, 1989). Figure 1 (a) shows one seismogram (for the azimuth of 45°) as an example. The symmetry-axis azimuth is \( \phi_0 = 60^\circ \). In reality, it was the earth model of 11 layers for the Urubchen-Tokhomo Area (Eastern Siberia), in which the HTI layer was the tenth (460 m-thick Riphean carbonates). Here, we perform AVOA analysis for the lower reflecting boundary of the HTI layer at the depth of 2675 m, marked by an arrow in Fig. 1 (a) ; note negative (inverse) change of the impedance over the boundary, from Table 1, that is \( A < 0 \).

We estimate the angle \( \phi_0 \) with the accuracy of 1.3%, that is \( \phi_0 = 60.8^\circ \). Then, taking this value of \( \phi_0 \) we validate the algorithm for distinguishing the symmetry-axis direction by calculating the parameter \( \Delta \varepsilon \) from (13) \( ( \Delta \varepsilon = -3.5 ) \) and that yields \( \text{sign}(A) \Delta \varepsilon > 0 \), as it should be in the case for the symmetry axis, equation (15).

Then, we tried the field data for the algorithm application. The field data contain 12 seismograms from 12 azimuth sectors, Figure 2 (a), for a superbin 13x13 bins that was acquired at the Vancor Area (Eastern Siberia, Russia) in 2004. The target reflection is the top of the HTI layer (overlain by an isotropic overburden) at the depth of 3.4 km marked by an arrow in Fig.2(a). The analysis of azimuthal variation in the AVO gradient provides two principle directions of symmetry, these are 8° and 98° in Figure 2(b). From these two we have distinguished the azimuth of 98° as the symmetry-axis direction. We use our criterion with taking into account the coefficient \( A_j \) of negative value, which should be negative due to the negative impedance change across the reflection boundary, \( \Delta Z < 0 \) (that was estimated from the sonic logs). We substitute calculated values of \( C_j \) \((j=1,..., 12)\) and \( \phi_0 = 8^\circ \) into equations (13), and that yields the parameter \( \Delta \varepsilon = -14.5 \). Following equation (14), the condition for the symmetry axis at the boundary isotropic medium - HTI medium requires \( \text{sign}(A) \Delta \varepsilon < 0 \), but in this case \( \text{sign}(A) = -1 \), and therefore \( \text{sign}(A) \Delta \varepsilon > 0 \). So we conclude that \( \phi_0 = 8^\circ \) is the fracture-strike direction, and the symmetry-axis azimuth is 98°.

Discussion and conclusions

For some cases, the problem of ambiguity in fracture-orientation detection may be resolved by analyzing the AVO-gradient dependence on \( \phi \), equation (3). As shown by Hall & Kendall (2000) and Zheng et al. (2004), if the sign of \( B_{\text{ani}} \) is priory known, then it is possible to resolve the problem. However, in general case, the sign of \( B_{\text{ani}} \) may be arbitrary. For many anisotropic media \( B_{\text{ani}} > 0 \), but also it occurs \( B_{\text{ani}} < 0 \) (Chichinina et al., 2003) that can not be priory known.

We resolve the problem of ambiguity by analysis of the sign change in the parameter \( \Delta \varepsilon \), which is the second coefficient in the high-angle term \( C \) in Ruger’s equation. Certainly, we realize that it is really hard task to estimate a reliable value of Ruger’s high-angle term \( C \) from real field

Table 1. Model input parameters

<table>
<thead>
<tr>
<th>Layer</th>
<th>Vertical velocities [m/s]</th>
<th>Density [g/cm³]</th>
<th>Anisotropy parameters</th>
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<td></td>
<td>( V_p )</td>
<td>( V_s )</td>
<td>( \rho )</td>
</tr>
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<td>HTI</td>
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<td>4114</td>
<td>2.8</td>
</tr>
<tr>
<td>Isotrop.</td>
<td>3700</td>
<td>1500</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Figure 1.
(a): The synthetic seismogram for the source-to-receiver-line azimuth of 45°.
(b): Azimuthal variation of the normalized AVO gradient \( B/A \).
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data, and moreover to extract the reliable value of the parameter \( \Delta \epsilon \) from the term \( C \). However, actually, only the sign of \( \Delta \epsilon \) just is required for the criterion, but not the value of \( \Delta \epsilon \). For the determining the sign of \( \Delta \epsilon \) we use equations (13), in which we include only the reliable values of coefficients \( C_j \) for each sector \( j \). The unreliable values of \( C_j \) are not taken into account; for example, the incomprehensibly large value of \( C_j \) from the azimuth sector of \( 7^\circ \) \((j=1)\) is omitted (see the example of the field-data application).

The corresponding AVO-gradient value \( B_j \) (for the first sector, \( j=1 \)) also exhibits an unreliable value and therefore it is excluded from the \( \cos 2(\phi - \phi_b) \) -fitting for the variation \( B(\phi) \) in Fig. 2 (b). This point was eliminated from the total amount of 12 points in the plot and so there are only 11 points remained for the fit.

Following equations 1 and (7)-(8), it is obvious, that the reliable estimate of \( C_j \) is linked to the reliable \( B_j \) -estimate. So, by selection the reliable \( B_j \) -estimate, we can provide with the reliable \( C_j \) -estimate. To select the correct (reliable) \( B_j \) -estimate, we perform the linear fitting \( y = A + Bx \) (where \( x = \sin^2 \theta \)), additionally to the polynomial fitting \( y = A + Bx + Cx^2 \), which is carried out at the first step, equations (7)-(8). Figure 2 (b) shows the trend of the polynomial fit similar to the trend of the linear fit, that is the only difference of 2.2° in the \( \phi_b \) -estimate.

We believe that the algorithm is applicable to field data by using the controlled values of \( C_j \), that are consistent with the correct values of \( B_j \).

Figure 2. (a): CMP gathers for twelve azimuth sectors of \( 15^\circ \)-width; (b): The values \( B_j / A_j \) \((j=1,...,11)\) estimated from the polynomial fit (marked by the black squares) and from the linear fit (the circles); AVO-gradient variation \( B(\phi) \) (solid line and dashed line, respectively) from the \( \cos 2(\phi - \phi_b) \) -fitting (the fit functions are given within the plot).

Acknowledgments

We thank the Institute of Geophysics in Novosibirsk, Russia for access to the synthetic seismograms and especially V. Kuznetsov (VNIIGEOFIZIKA, Moscow) who made the modeling by himself. We are grateful to I. Korsunov and Yu. Kornelev ("Enisey-Geofizika", Krasnoyarsk) for the field data and the processing with ProMAX. We are grateful to S. Gorshkalev and W. Karsten from the Institute of Geophysics for helpful discussions and consultation on azimuth-sectoring of 3D-data for the analysis AVOA.

References


EDITED REFERENCES
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REFERENCES